Lecture no.1

Basic Concepts about Nonideal Plasma

Introduction

It is well known that at low densities plasma can be considered as a mixture of ideal gases of electrons, atoms, and ions. In this case the particles move along straight lines, and sometimes collide with other particles. With increasing of plasma density, the average distances between particles decrease and particle's interacting time increases, therefore, the average potential energy increases. If this energy gets to be comparable with average kinetic energy of thermal motion, i.e., $\bar{U} \approx \bar{E}_{kinetic}$, the plasma becomes **nonideal.** It should be noted that properties of such plasma cannot be described by traditional methods of theoretical physics. The interaction between particles in fully ionized plasma can be described by long-range Coulomb potential. In the case of complex plasma consisting of electrons, ions, atoms, molecules, clusters etc., different interaction potentials should be used.

Interparticle Interactions and Criteria of Nonideality

The ratio between the average interaction potential energy of particles and the mean thermal energy $k_B T$ is used as a criterion of nonideality of a plasma. For nondegenerate singly ionized plasma this condition can be written by coupling (nonideality) parameter Γ :

$$\Gamma = \frac{e^2}{a k_B T} \quad , \tag{1}$$

where a is the average distance between particles and related to the plasma density by the following simple relation:

$$(4/3)\pi n_e a^3 = 1 \quad . \tag{2}$$

In the case of multiple ionized plasma the different nonideality parameters for ion–ion, ion–electron, and electron–electron interactions should be used. For example, in a fully ionized plasma with ions having charge number Z we have the following relations:

$$\Gamma_{ZZ} = \frac{Z^{5/3}e^2}{ak_BT} = Z^{5/3}\Gamma_{ee};$$
$$\Gamma_{Ze} = \frac{Z^{2/3}e^2}{ak_BT} = Z^{2/3}\Gamma_{ee}; \qquad (3)$$
$$\Gamma_{ee} = \frac{e^2}{ak_BT}.$$

It should be noted that coupling parameters (1) and (3) can be applied for semiclassical dense plasma. For describing of classical plasma the following **nonideality parameter** is usually used:

$$\gamma = \frac{Z_1Z_2e^2}{r_Dk_BT}, \qquad (4)$$

where r_D is the Debye screening radius. Thus we can consider the following types of plasma by above mentioned parameters:

- Ideal plasma (at $\Gamma,\gamma \ll 1$).
- Weakly nonideal plasma (at $\Gamma,\gamma < 1$).
- Nonideal plasma (at $\Gamma,\gamma \geq 1$).
- Strongly nonideal (coupled) plasma (at $\Gamma,\gamma \gg 1$).

To determine of the condition of classicality we should compare the characteristic distance between particles with the thermal electron wavelength $\lambda_e = h/(2m_ek_BT)$. Since the minimal characteristic radius of the ion–electron interaction is Ze^2/k_BT, the condition of classicality can be written as

$$\lambda_e \ll \frac{Ze^2}{k_BT}. \qquad (5)$$

The condition of classicality can be also written in terms of the degeneration parameter ξ:

$$\xi = \frac{\varepsilon_F}{k_B T} \ll 1 \quad , \tag{6}$$

here $\varepsilon_F = \left(3\pi^2 n_e\right)^{2/3} h^2 / 2m$ is the Fermi energy. Sometimes another degeneration parameter $\theta = 1/\xi$ can be also used for describing of semiclassical plasma's properties.

Notice that further compression of the plasma causes an increase of nonideality, but up to a certain value (limit) only, because at $n_e \lambda_e^3 \sim 1$ with increasing density degeneracy of electrons occurs. For example, in metals $n_e \sim 10^{23} cm^{-3}$ and electrons are degenerate at $T \le 10^5 K$, i.e., almost always. With increasing of plasma density the Fermi energy can be chosen as a kinetic energy scale. Therefore, the quantum criterion of ideality has the following form:

$$\Gamma_q = e^2 n_e^{1/3} / \varepsilon_F \ll 1 \, . \tag{7}$$

Since $\varepsilon_F \sim n_e^{2/3}$ we can conclude that $\Gamma_q \sim n_e^{1/3} / \varepsilon_F \sim n_e^{-1/3}$, i.e., the quantum criterion parameter Γ_q decreases with increasing electron density. Consequently, the degenerate electron plasma becomes more ideal with compression. Notice that, at higher densities, only electrons can be considered as an ideal Fermi gas, whereas the ion component is nonideal.

As a dimensionless density parameter r_S the ratio between average interparticle distance a and the Bohr radius a_B is used $r_S = a / a_B$, where $a_B = h / me^2 \approx 0,5 \cdot 10^{-8} cm$.

Screening of Charged Particle's Field in Plasma

Due to the long–range character of the Coulomb potential the manyparticle interactions at large distances are important. The potential created by the selected test particle and its plasma environment is the well known Debye potential:

$$\varphi = \frac{e^2}{r} e^{-r/r_D} \quad , \tag{8}$$

where k corresponds to different charged plasma species and r_D is the Debye screening radius:

$$r_D = \left(4\pi e^2 \sum_k Z_k^2 n_k \Big/ k_B T \right)^{-1/2} . \tag{9}$$

According to (4) the criterion of ideality for singly charged plasma can be written as

$$\gamma = \frac{e^2}{r_D k_B T} \ll 1 \quad . \tag{10}$$

Let us introduce the number of electrons in the Debye sphere $N_D = (4/3)\pi n_e r_D^3$. Then the criterion (10) can be expressed in terms of N_D as $\gamma = (3\Gamma^3)^{1/2} = (3N_D)^{-1}$. For ideal plasma we have condition $N_D \gg 1$.

If the electrons of plasma are degenerate, i.e., $n_e \lambda_e^3 \gg 1$, the screening length by degenerate electrons is defined by the Thomas–Fermi radius:

$$r_{TF} = (\pi / 3n_e)\sqrt{h^2 / 4me^2} \quad . \tag{11}$$

In a two–component electron–ion plasma, in which the electrons are degenerate but ions are classical, the screening radius of the test charge is defined by the following expression:

$$r^{-2} = (r_{TF}^e)^{-2} + (r_D^i)^{-2} \approx (r_D^i)^{-2} \quad . \tag{12}$$

Quantum Effects in Interparticle Interactions

At small distances (when average distance between particles is approximately equal to the thermal wave-length, i.e., $a \sim \lambda_e$) we have to take into account quantum effects (for instance, diffraction and symmetry effects). These effects lead to the formation of atoms and molecules and play an important role. Taking into account these effects eliminates the divergencies at small distances between particles.

For adequate taking into account of quantum effects at small distances the Slater sum and the Boltzmann factor should be jointly applied. It is known that the probability density of finding two particles at a distance r in classical statistics is proportional to the Boltzmann factor $\exp\left(-\Phi(r)/k_B T\right)$, here $\Phi(r)$ is the interaction potential between two particles. In quantum physics such probability is defined by the Slater sum:

$$S_2(r,T) = 2\lambda_e^3 \sum_n \Psi_n^* \exp\left(-E_n / k_B T\right) \Psi_n, \tag{13}$$

where Ψ_n and E_n are the wavefunctions and the corresponding eigenvalues of the energy of two particles, respectively, and the summation in (13) is performed over all states of discrete and continuous spectra.

Let us define the pseudopotential $\tilde{\Phi}(r,T)$ as a potential giving in the classical case the same particle distribution in space as the potential $\Phi(r)$ gives in the quantum case, i.e.

$$\tilde{\Phi}(r,T) = -k_B T \ln S_2(r,T) . \tag{14}$$

Notice that the pseudopotential $\tilde{\Phi}(r,T)$ has the limiting value at $r=0$ and coincides with $\Phi(r)$ in the limit $T \to \infty$. At large distances ($r \to \infty$) $\tilde{\Phi}(r,T)$ has a Coulomb-like asymptotic dependence.

From equation (14) at $T \to \infty$ and $n_e, n_i \to \infty$ the following expression for effective potential is obtained (C.Deutsch e.a., 1980):

$$\Phi_{\alpha\beta}(r) = \frac{e^2}{r}\left(1 - \exp\left(-\frac{r}{\lambda_{\alpha\beta}}\right)\right) + \delta_{\alpha\beta}\delta_{e\alpha} \ln(2) k_B T \exp\left(-\frac{r^2}{\pi \ln(2) \lambda_{ee}^2}\right) \tag{15}$$

Taking into Account both Quantum and Screening Effects

It should be noted that even in a rarefied plasma, when $\gamma \ll 1$, one cannot directly apply the formulas of ideal gas theory for describing the thermodynamic and transport properties of the plasma. Some quantities such as the second virial coefficient or the mean free path are diverging due to the specific character of the Coulomb interaction. It is known that the Coulomb potential has a long range character at large distances and an infinite divergence at small distances.

The divergence at small distances is eliminated by taking into account of quantum diffraction and symmetry effects. The divergencies of physical quantities at large distances can be eliminated by taking into account the effect of charge screening in plasma.

Notice that diffraction effect is related to the de Broglie waves of microparticles and symmetry effect corresponds to the Pauli exclusion principle.

Consequently, in dense semiclassical plasma the collective (screening) and quantum-mechanical effects play an important role in the studies of thermodynamic and kinetic properties of the system. In general case these potentials contain quantum diffraction effects at short distances, as well as screening effects for large distances (T.Ramazanov e.a., 2002):

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta e^2}{\sqrt{1-4\lambda_{\alpha\beta}^2/r_D^2}}\left(\frac{e^{-Ar}}{r} - \frac{e^{-Br}}{r}\right) \tag{16}$$

for electron-electron and electron-ion interactions and here

$$A^2 = \frac{1}{2\lambda_{\alpha\beta}^2}\left(1-\sqrt{1-\lambda_{\alpha\beta}^2/r_D^2}\right); \quad B^2 = \frac{1}{2\lambda_{\alpha\beta}^2}\left(1+\sqrt{1-\lambda_{\alpha\beta}^2/r_D^2}\right).$$

For describing of ion-ion interactions we have the following expression (T.Ramazanov e.a., 2010) :

$$\Phi_{ii}(r) = \frac{Z_i Z_i e^2}{\sqrt{1-4\lambda_{ei}^2/r_D^2}}\left(\lambda_{ei}^2 B^2 \frac{e^{-Ar}}{r} - \lambda_{ei}^2 A^2 \frac{e^{-Br}}{r}\right) \tag{17}$$

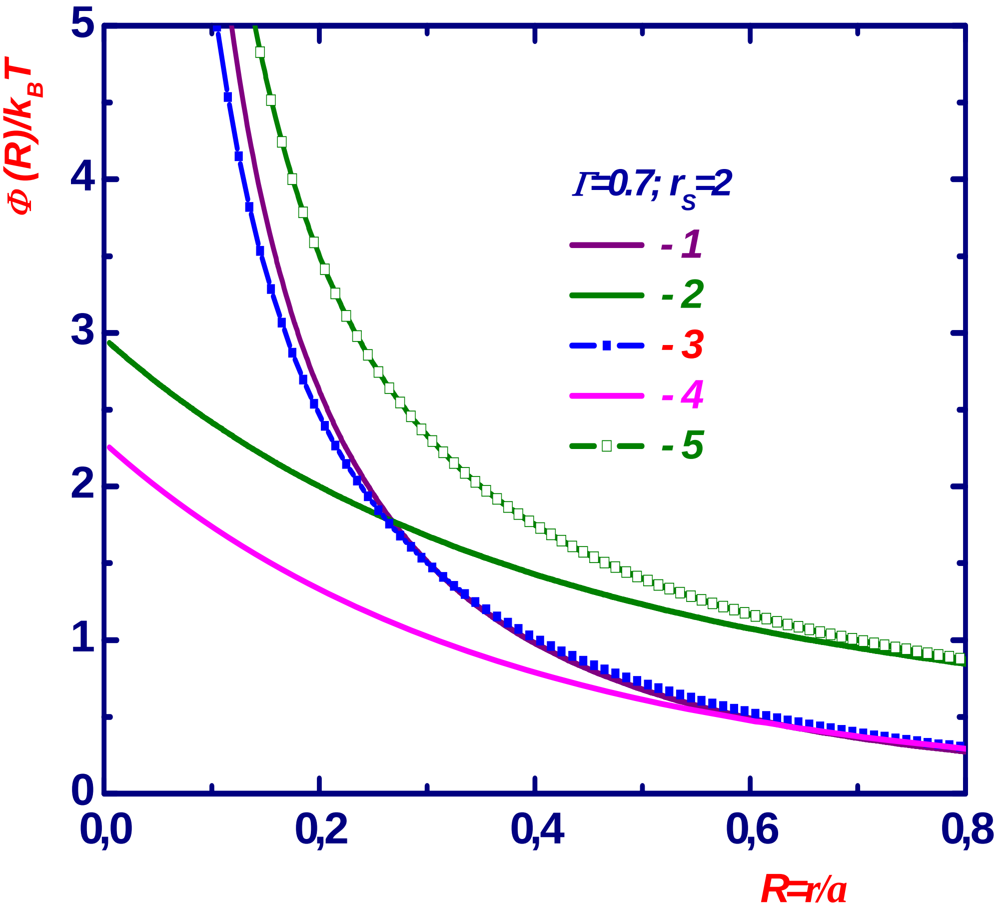

Figure 1. Effective potentials for fully ionized semiclassical plasma.

1 – The Debye potential; 2 – The Deutsch potential;

3 – (T.Ramazanov etc., ion-ion)

4 – (T.Ramazanov etc., e-e, e-i)

5 – The Deutsch potential for i-i interaction;